Dipole and quadrupole moments of W_L and W_R bosons in the left-right supersymmetric model

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We perform a complete and detailed analysis of the anomalous dipole $\Delta\kappa_\gamma$ and quadrupole moments ΔQ_γ of both W_L and W_R bosons in a supersymmetric left-right model. After a discussion of the mechanism of symmetry breaking in the model, we give complete expressions for the *CP*-conserving static moments of W_L and W_R. We then perform a numerical analysis of the charged vector bosons moments as functions of the parameters in the soft-breaking supersymmetric variables m_0, $M_{1/2}$, μ, A_0, A_{LR}, and $\tan\beta$. The values for $\Delta\kappa_\gamma$ and ΔQ_γ for the W_L boson can vary from the minimal supersymmetric standard model values. Although these couplings are unlikely to be observed by further analysis of the data at CERN LEP2, they might reach the sensitivity of CERN LHC, and will be observable at a future New Linear Collider, which should reach precision of $\mathcal{O}(10^{-3})$ to $\mathcal{O}(10^{-4})$. We also present for the first time complete analytical expressions for the static moments of the W_R boson, including supersymmetric contributions, and discuss possible numerical values for a range of supersymmetric parameters.

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I. INTRODUCTION

Triple or quadruple vector boson interactions have always been considered theoretically interesting because they test the non-Abelian structure of the standard model (SM). The three charged boson self-interaction vertices correspond to γWW, ZWW interactions, for which couplings are specified in the bosonic part of the Lagrangian. Although present measurements of the vector boson couplings at the CERN e^+e^- collider LEP and SLAC Linear Collider (SLC) confirm the SM predictions to a high degree of accuracy, there remain still some theoretical problems if one looks at higher energies. One way to reconcile this is to consider the SM as an effective low-energy theory and assume that new physics exists at a higher energy scale, and that it induces deviations of physical observables from the SM predictions. Present experiments at Fermilab Tevatron and LEP2 have relatively low sensitivity and are unable to explore these gauge couplings to the required accuracy for detecting new physics; however, it is hoped that significantly improved facilities such as the New Linear Collider (NLC) would allow further tests.

The SM predicts the anomalous dipole and quadrupole moments of the W_L boson to be zero at tree level, $\Delta\kappa_\gamma = \Delta Q_\gamma = 0$ [1]. Higher order corrections have been evaluated and modify these results by small finite amounts [2]. It is important to compare these calculations with exact estimates in models beyond the SM. As a scenario for new physics, supersymmetry has been seen as one of the most successful extensions of the standard model. Supersymmetry provides an elegant solution to the gauge hierarchy problem, and also appears to be a necessary ingredient towards embedding the standard model in a single gauge symmetry. Supersymmetric grand unified theories provide gauge unification at a single scale, while theories without supersymmetry do not. Although theoretical studies of supersymmetric theories exist, most concentrate on the minimal extension of the supersymmetric standard model (MSSM). Recent data on solar [3] and atmospheric [4] neutrino oscillations gives clear indications that neutrinos have mass and therefore MSSM as a symmetry is insufficient. At least, it would have to be augmented by right-handed singlet neutrinos, or else one would have to abandon *R*-parity conservation, defined as $R = (-1)^{3(B-L)+2S}$. Several studies on supersymmetric scenarios beyond the MSSM concentrate on the fermion sector, rather than the bosonic sector.

In this work we propose to analyze the static properties of the charged bosons in a simple extension of the MSSM incorporating left-right symmetry. The left-right supersymmetric model (LRSUSY) is perhaps the most natural extension of the minimal supersymmetric model [5–8]. Left-right supersymmetry is based on the group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$, which would then break spontaneously to $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ [5]. LRSUSY was originally seen as a natural way to suppress rapid proton decay and as a mechanism for providing small neutrino masses [7]. Besides being a plausible symmetry itself, the LRSUSY model has the added attractive feature that it can be embedded in a supersymmetric grand unified theory such as SO(10) [9]. Support for left-right theories is provided by building realistic brane worlds from type I strings. This involves left-right supersymmetry, with supersymmetry broken either at the string scale $M_{\mathrm{SUSY}} \approx 10^{10}$–$10^{12}$ GeV, or at $M_{\mathrm{SUSY}} \approx 1$ TeV, the difference having implications for gauge unification [10]. A different pattern of symmetry breaking can occur if the gauge symmetry is broken down through orbifold compactification from five to four dimensions, scenarios in which W_R emerges as a Kaluza-Klein excited state [11,12].

In what follows we shall concentrate on evaluating the anomalous dipole and quadrupole moments of both W_L and W_R bosons in the left-right supersymmetric model. We do not assume that LRSUSY is obtained by spontaneous symmetry breaking from a SUSYGUT scenario, although this is not ruled out; but we want to keep our model as general as

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possible. We choose a Higgs sector which supports the see-saw mechanism. We study the breaking of the LRSUSY model to the SM and its consequences in the bosonic sector. We give Feynman rules and explicit analytical expressions for the anomalous dipole and quadrupole moments for both W_L and W_R, before performing a numerical estimate for these quantities, using symmetry breaking and renormalization group equations constraints.

Our work is organized as follows. In Sec. II we describe the LRSUSY model and the pattern of symmetry breaking to the SM. In Sec. III we give the complete analytical expressions for the anomalous dipole and quadrupole moments. In Sec. IV we perform a detailed numerical analysis of the static moments and identify the parameters the static moments are most sensitive to. We summarize and conclude in Sec. V.

II. DESCRIPTION OF THE LRSUSY MODEL

The minimal supersymmetric left-right model is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The matter field of this model consists of three families of quark and lepton chiral superfields with the following transformations under the gauge group:

$$Q=\begin{pmatrix} u \\ d \end{pmatrix} \sim \left(3,2,1,\frac{1}{3}\right), \quad Q^c=\begin{pmatrix} d^c \\ u^c \end{pmatrix} \sim \left(3^*,1,2,-\frac{1}{3}\right),$$

$$L=\begin{pmatrix} \nu \\ e \end{pmatrix} \sim (1,2,1,-1), \quad L^c=\begin{pmatrix} e^c \\ \nu^c \end{pmatrix} \sim (1,1,2,1), \tag{1}$$

where the numbers in the brackets denote the quantum numbers under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The Higgs sector consists of the bidoublet and triplet Higgs superfields:

$$\Phi_1=\begin{pmatrix} \Phi_{11}^0 & \Phi_{11}^+ \\ \Phi_{12}^- & \Phi_{12}^0 \end{pmatrix} \sim (1,2,2,0),$$

$$\Phi_2=\begin{pmatrix} \Phi_{21}^0 & \Phi_{21}^+ \\ \Phi_{22}^- & \Phi_{22}^0 \end{pmatrix} \sim (1,2,2,0),$$

$$\Delta_L=\begin{pmatrix} \frac{1}{\sqrt{2}}\Delta_L^- & \Delta_L^0 \\ \Delta_L^{--} & -\frac{1}{\sqrt{2}}\Delta_L^- \end{pmatrix} \sim (1,3,1,-2),$$

$$\delta_L=\begin{pmatrix} \frac{1}{\sqrt{2}}\delta_L^+ & \delta_L^{++} \\ \delta_L^0 & -\frac{1}{\sqrt{2}}\delta_L^+ \end{pmatrix} \sim (1,3,1,2), \tag{2}$$

$$\Delta_R=\begin{pmatrix} \frac{1}{\sqrt{2}}\Delta_R^- & \Delta_R^0 \\ \Delta_R^{--} & -\frac{1}{\sqrt{2}}\Delta_R^- \end{pmatrix} \sim (1,1,3,-2),$$

$$\delta_R=\begin{pmatrix} \frac{1}{\sqrt{2}}\delta_R^+ & \delta_R^{++} \\ \delta_R^0 & -\frac{1}{\sqrt{2}}\delta_R^+ \end{pmatrix} \sim (1,1,3,2).$$

The bidoublet Higgs superfields appear in all LRSUSY and serve to implement the $SU(2)_L \times U(1)_Y$ symmetry breaking and to generate a Cabibbo-Kobayashi-Maskawa mixing matrix. Supplementary Higgs representations are needed to break left-right symmetry spontaneously: either doublets or triplets would achieve this, but the triplet Higgs Δ_L, Δ_R bosons have the advantage of supporting the seesaw mechanism. Since the theory is supersymmetric, additional triplet superfields δ_L, δ_R are needed to cancel triangle gauge anomalies in the fermionic sector. The most general superpotential involving these superfields is

$$\begin{aligned} W=&\mathbf{Y}_Q^{(i)} Q^T \Phi_i i\tau_2 Q^c + \mathbf{Y}_L^{(i)} L^T \Phi_i i\tau_2 L^c + i(\mathbf{Y}_{LR} L^T \tau_2 \delta_L L \\ &+ \mathbf{Y}_{LR} L^{cT} \tau_2 \Delta_R L^c) + \mu_{LR}[\mathrm{Tr}(\Delta_L \delta_L + \Delta_R \delta_R)] \\ &+ \mu_{ij} \mathrm{Tr}(i\tau_2 \Phi_i^T i\tau_2 \Phi_j) + W_{NR}, \end{aligned} \tag{3}$$

where W_{NR} denotes (possible) nonrenormalizable terms arising from higher scale physics or Planck scale effects [13]. The presence of these terms insures that, when the SUSY breaking scale is above M_{W_R}, the ground state is R-parity conserving [14]. In addition, the potential also includes well-known F-terms, D-terms as well as soft supersymmetry breaking:

$$\begin{aligned} \mathcal{L}_{\mathrm{soft}}=&[\mathbf{A}_Q^i \mathbf{Y}_Q^{(i)} \tilde{Q}^T \Phi_i i\tau_2 \tilde{Q}^c + \mathbf{A}_L^i \mathbf{Y}_L^{(i)} \tilde{L}^T \Phi_i i\tau_2 \tilde{L}^c \\ &+ i\mathbf{A}_{LR} \mathbf{Y}_{LR}(\tilde{L}^T \tau_2 \delta_L \tilde{L} + L^{cT} \tau_2 \Delta_R \tilde{L}^c) + m_\Phi^{(ij)2} \Phi_i^\dagger \Phi_j] \\ &+ [(m_L^2)_{ij} \tilde{l}_{Li}^\dagger \tilde{l}_{Lj} + (m_R^2)_{ij} \tilde{l}_{Ri}^\dagger \tilde{l}_{Rj}] - M_{LR}^2[\mathrm{Tr}(\Delta_R \delta_R) \\ &+ \mathrm{Tr}(\Delta_L \delta_L) + \mathrm{H.c.}] - [B\mu_{ij} \Phi_i \Phi_j + \mathrm{H.c.}]. \end{aligned} \tag{4}$$

Symmetry breaking and gauge boson masses

As in the standard model, the symmetry is broken spontaneously to $U(1)_{em}$. In order to preserve $U(1)_{em}$ gauge invariance, only the neutral Higgs fields acquire nonzero vacuum expectation values (VEVs). These values are

$$\langle \Phi_1 \rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_1' e^{i\omega_1} \end{pmatrix}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} \kappa_2' e^{i\omega_2} & 0 \\ 0 & \kappa_2 \end{pmatrix},$$

$$\langle \Delta_L \rangle = \begin{pmatrix} 0 & v_{\Delta_L} \\ 0 & 0 \end{pmatrix},$$

$$\langle\ \delta_L\rangle=\begin{pmatrix}0 & 0\\ v_{\delta_L} & 0\end{pmatrix},\ \langle\ \Delta_R\rangle=\begin{pmatrix}0 & v_{\Delta_R}\\ 0 & 0\end{pmatrix},$$

$$\langle\ \delta_R\rangle=\begin{pmatrix}0 & 0\\ v_{\delta_R} & 0\end{pmatrix}.$$

There are three different stages of symmetry breakdown. At the first stage only discrete parity is broken:

$$\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}\times P \xrightarrow{M_P} \mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}. \quad (5)$$

This breaking occurs at $M_P\equiv M_{\mathrm{parity}}$. As a result $g_L\neq g_R$, however, no gauge boson mass is generated. In the second stage of symmetry breaking, due to $\langle\Delta_R\rangle\neq 0$, $\langle\delta_R\rangle\neq 0$:

$$\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}\xrightarrow{M_R}\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y. \quad (6)$$

At this stage, the right-handed gauge bosons, W_R and Z_R acquire masses proportional to v_{Δ_R}, v_{δ_R} and become much heavier than the usual (left-handed) neutral gauge bosons W_L and Z_L:

$$M_{W_R}=\frac{v_{\Delta_R}g_R}{\sqrt{2}},$$

$$M_{Z_R}=v_{\Delta_R}\sqrt{g_R^2+4g_V^2}=\frac{v_{\Delta_R}g_R}{\cos\phi}, \quad (7)$$

where $\tan\phi=2g_V/g_R$ is the mixing angle in the right-handed sector. One can assume for simplicity that $M_P=M_R$, without consequences for vector boson masses. After this stage, the model has the same symmetries as the MSSM. The final stage of symmetry breakdown takes place at electroweak scales M_L, and

$$\mathrm{SU}(2)_L\times\mathrm{U}(1)_{B-L}\xrightarrow{M_L}\mathrm{U}(1)_{em} \quad (8)$$

through bidoublet VEVS κ_1, $\kappa_2\neq 0$. Bosonic masses at this stage are

$$M_{W_L}=\frac{1}{2}g_L\sqrt{\kappa_1^2+\kappa_2^2},$$

$$M_{Z_L}=\frac{1}{2}\frac{g_L}{\cos\theta_W}\sqrt{\kappa_1^2+\kappa_2^2}, \quad (9)$$

$$M_\gamma=0,$$

where $\tan\theta_W=g_R\sin\phi/g_L$ is the weak mixing angle. Left-right symmetry ($g_L=g_R$) would mean $\sin\phi=\tan\theta_W$. Note the difference in the right-handed and left-handed mass ratio: while for the left-handed bosons we have

$$\frac{M_{W_L}}{M_{Z_L}}=\cos\theta_W,$$

for the right-handed counterparts we get

$$\frac{M_{W_R}}{M_{Z_R}}=\frac{\cos\phi}{\sqrt{2}}.$$

The reason is that the right-handed gauge bosons get their mass through the VEV of a triplet field, while the left-handed fields receive their mass through bidoublet field VEVS. The mixing between the bare and physical gauge bosons is given by

$$\begin{pmatrix}Z_\mu^L\\ A_\mu\\ Z_\mu^R\end{pmatrix}=\begin{pmatrix}\cos\theta_W & -\sin\theta_W\tan\theta_W & -\tan\theta_W\sqrt{\cos(2\theta_W)}\\ \sin\theta_W & \sin\theta_W & \sqrt{\cos(2\theta_W)}\\ 0 & \dfrac{\sqrt{\cos(2\theta_W)}}{\cos\theta_W} & -\tan\theta_W\end{pmatrix}\begin{pmatrix}W_\mu^{0L}\\ W_\mu^{0R}\\ V_\mu\end{pmatrix}. \quad (10)$$

In addition, supersymmetry can be broken at any scale between M_R and M_L.

The above presentation does not elaborate on the possible values for M_R. There is a close relationship between the method used to break LR symmetry and the mass of the right-handed gauge boson. If one enlarges the Higgs sector of the minimal LRSUSY model by introducing a parity-odd Higgs singlet, coupled appropriately to the triplet fields, this would give rise to a LR breaking scale around 10 TeV [14]. An alternative is to either consider higher dimensional operators [15], or to introduce an intermediate $B-L$ breaking scale [16], both of which would yield a large M_R. Although experiments still allow for a light M_R, the success of the standard model and seesaw mechanism for neutrino masses hint at a larger M_R.

III. MAGNETIC DIPOLE AND QUADRUPOLE MOMENTS: ANALYTICAL RESULTS

The static moments of $W_L^\pm$ and $W_R^\pm$ are obtained from the effective vertex function $\Gamma^{\mu\lambda\rho}$ calculated for the interactions between a photon field A and two $W_L(W_R)$ boson fields. The anomalous magnetic moment is denoted by $\kappa_\gamma-1$ and the anomalous quadrupole moment by ΔQ_γ. The most general

TABLE I. Contributions to the magnetic dipole moment of W_L to one-loop order.

Particles in the loop	τ	σ	$\kappa_\gamma - 1$
$W_L W_L A$	1	0	$\frac{5}{3}\frac{\alpha}{\pi}$
$W_L W_L Z_L$	1	$\frac{M_{Z_L}}{M_{W_L}}$	$\frac{g_L^2}{16\pi^2}\left(\frac{20}{3\sigma^2} - \frac{5}{6} + \int_0^1 dz \frac{z^4/2 + 5z^3 - 18z^2 + 16z - 8}{\sigma^2(1-z) + z^2}\right)$
$\tilde{u}_{Ln}\tilde{u}_{Ln}\tilde{d}_{Lm}$	$\frac{M_{\tilde{u}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{d}_{Lm}}}{M_{W_L}}$	$-\frac{N_c g_L^2}{16\pi^2}\left(\frac{2}{3}\right)\tilde{X}^*_{mn}\tilde{X}_{mn}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 - z^2(\tau_n^2 - 1)}{\sigma_m^2(1-z) + z^2 + z(\tau_n^2 - 1)}\right)$
$\tilde{d}_{Ln}\tilde{d}_{Ln}\tilde{u}_{Lm}$	$\frac{M_{\tilde{d}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{u}_{Lm}}}{M_{W_L}}$	$-\frac{N_c g_L^2}{16\pi^2}\left(-\frac{1}{3}\right)\tilde{X}^*_{mn}\tilde{X}_{mn}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 - z^2(\tau_n^2 - 1)}{\sigma_m^2(1-z) + z^2 + z(\tau_n^2 - 1)}\right)$
$\tilde{e}_{Ln}\tilde{e}_{Ln}\tilde{\nu}_{Lm}$	$\frac{M_{\tilde{e}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{\nu}_{Lm}}}{M_{W_L}}$	$-\frac{g_L^2}{16\pi^2}(-1)\tilde{Y}^*_{mn}\tilde{Y}_{mn}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 - z^2(\tau_n^2 - 1)}{\sigma_m^2(1-z) + z^2 + z(\tau_n^2 - 1)}\right)$
$u_{Ln}u_{Ln}d_{Lm}$	$\frac{M_{u_{Ln}}}{M_{W_L}}$	$\frac{M_{d_{Lm}}}{M_{W_L}}$	$-\frac{N_c g_L^2}{16\pi^2}\left(\frac{2}{3}\right)X^*_{mn}X_{mn}\left(\frac{1}{6} - \int_0^1 dz \frac{z^4 - z^3 + z^2 + z^2(\tau_n^2 - 1)}{\sigma_m^2(1-z) + z^2 + z(\tau_n^2 - 1)}\right)$
$d_{Ln}d_{Ln}u_{Lm}$	$\frac{M_{d_{Ln}}}{M_{W_L}}$	$\frac{M_{u_{Lm}}}{M_{W_L}}$	$-\frac{N_c g_L^2}{16\pi^2}\left(-\frac{1}{3}\right)X^*_{mn}X_{mn}\left(\frac{1}{6} - \int_0^1 dz \frac{z^4 - z^3 + z^2 + z^2(\tau_n^2 - 1)}{\sigma^2(1-z) + z^2 + z(\tau_n^2 - 1)}\right)$
$e_{Ln}e_{Ln}\nu_{Lm}$	$\frac{M_{e_{Ln}}}{M_{W_L}}$	$\frac{M_{\nu_{Lm}}}{M_{W_L}}$	$-\frac{g_L^2}{16\pi^2}(-1)Y^*_{mn}Y_{mn}\left(\frac{1}{6} - \int_0^1 dz \frac{z^4 - z^3 + z^2 + z^2(\tau_n^2 - 1)}{\sigma_m^2(1-z) + z^2 + z(\tau_n^2 - 1)}\right)$
$\tilde{\chi}_j^+\tilde{\chi}_j^+\tilde{\chi}_k^0$	$\frac{M_{\tilde{\chi}_j^+}}{M_{W_L}}$	$\frac{M_{\tilde{\chi}_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}(\lvert L_{kj}^L\rvert^2 + \lvert L_{kj}^R\rvert^2)\left(\frac{1}{6} - \int_0^1 dz \frac{z^4 - z^3 + z^2 + z^2(\tau_j^2 - 1)}{\sigma_k^2(1-z) + z^2 + z(\tau_j^2 - 1)}\right)$
			$-\frac{g_L^2}{8\pi^2}2\,\mathrm{Re}(L_{kj}^{L*}L_{kj}^R)\left(\tau_j\sigma_k\int_0^1 dz \frac{2z^2 - z}{\sigma_k^2(1-z) + z^2 + z(\tau_j^2 - 1)}\right)$
$H_j^{++}H_j^{--}H_k^+$ $j = 3,4;\ k = 1,\ldots,6$	$\frac{M_{H_j^{++}}}{M_{W_L}}$	$\frac{M_{H_k^+}}{M_{W_L}}$	$-2\frac{g_L^2}{8\pi^2}a_{2,kj}a_{2,kj}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 - z^2(\tau_j^2 - 1)}{\sigma_k^2(1-z) + z^2 + z(\tau_j^2 - 1)}\right)$
$H_j^+H_j^-H_k^{++}$ $j = 1,\ldots,6;\ k = 3,4$	$\frac{M_{H_j^+}}{M_{W_L}}$	$\frac{M_{H_k^{++}}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{2,kj}a_{2,kj}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 - z^2(\tau_j^2 - 1)}{\sigma_k^2(1-z) + z^2 + z(\tau_j^2 - 1)}\right)$
$H_j^+H_j^-H_k^0$ $j = 1,\ldots,6;\ k = 1,\ldots,8$	$\frac{M_{H_j^+}}{M_{W_L}}$	$\frac{M_{H_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{4,kj}a_{4,kj}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 - z^2(\tau_j^2 - 1)}{\sigma_k^2(1-z) + z^2 + z(\tau_j^2 - 1)}\right)$
$H_j^+H_j^-A_k^0$ $j = 1,\ldots,6;\ k = 1,\ldots,6$	$\frac{M_{H_j^+}}{M_{W_L}}$	$\frac{M_{A_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{6,kj}a_{6,kj}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 - z^2(\tau_j^2 - 1)}{\sigma_k^2(1-z) + z^2 + z(\tau_j^2 - 1)}\right)$
$W_L W_L H_j^0$ $j = 1,\ldots,6$	1	$\frac{M_{H_j^0}}{M_{W_L}}$	$\frac{g_L^4}{32\pi^2 M_{W_L}^2}a_{8,j}a_{8,j}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 + 4z^2}{\sigma_j^2(1-z) + z^2}\right)$
$W_R W_R H_j^0$ $j = 1,\ldots,6$	$\frac{M_{W_R}}{M_{W_L}}$	$\frac{M_{H_j^0}}{M_{W_L}}$	$\frac{g_L^2 g_R^2}{32\pi^2 M_{W_L}^2}a_{8,j}a_{8,j}\left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 + 4z^2}{\sigma_j^2(1-z) + z^2 + z(\tau - 1)}\right)$

TABLE II. Contributions to the magnetic dipole moment of W_R to one-loop order.

Particles in the loop	τ	σ	$\kappa_\gamma - 1$
$W_R W_R A$	1	0	$\frac{5}{3}\frac{\alpha}{\pi}$
$W_R W_R Z_L$	1	$\frac{M_{Z_L}}{M_{W_R}}$	$\frac{\sigma^2 e^2 \tan^2\theta_W}{16\pi^2}\left(\frac{20}{3\sigma^2}-\frac{5}{6}+\int_0^1 dz\,\frac{z^4/2+5z^3-18z^2+16z-8}{\sigma^2(1-z)+z^2}\right)$
$W_R W_R Z_R$	1	$\frac{M_{Z_R}}{M_{W_R}}$	$\frac{g_R^2}{8\pi^2}\left(\frac{20}{3\sigma^2}-\frac{5}{6}+\int_0^1 dz\,\frac{z^4/2+5z^3-18z^2+16z-8}{\sigma^2(1-z)+z^2}\right)$
$\tilde{u}_{Rn}\tilde{u}_{Rn}\tilde{d}_{Rm}$	$\frac{M_{\tilde{u}_{Rn}}}{M_{W_R}}$	$\frac{M_{\tilde{d}_{Rm}}}{M_{W_R}}$	$-\frac{N_c g_R^2}{16\pi^2}\left(\frac{2}{3}\right)\tilde{X}^*_{mn}\tilde{X}_{mn}\left(\frac{1}{3}+\int_0^1 dz\,\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{d}_{Rn}\tilde{d}_{Rn}\tilde{u}_{Rm}$	$\frac{M_{\tilde{d}_{Rn}}}{M_{W_R}}$	$\frac{M_{\tilde{u}_{Rm}}}{M_{W_R}}$	$-\frac{N_c g_R^2}{16\pi^2}\left(-\frac{1}{3}\right)\tilde{X}^*_{mn}\tilde{X}_{mn}\left(\frac{1}{3}+\int_0^1 dz\,\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{e}_{Rn}\tilde{e}_{Rn}\tilde{\nu}_{Rm}$	$\frac{M_{\tilde{e}_{Rn}}}{M_{W_R}}$	$\frac{M_{\tilde{\nu}_{Rm}}}{M_{W_R}}$	$-\frac{g_R^2}{16\pi^2}(-1)\tilde{Y}^*_{mn}\tilde{Y}_{mn}\left(\frac{1}{3}+\int_0^1 dz\,\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$u_{Rn}u_{Rn}d_{Rm}$	$\frac{M_{u_{Rn}}}{M_{W_R}}$	$\frac{M_{d_{Rm}}}{M_{W_R}}$	$-\frac{N_c g_R^2}{16\pi^2}\left(\frac{2}{3}\right)X^*_{mn}X_{mn}\left(\frac{1}{6}-\int_0^1 dz\,\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$d_{Rn}d_{Rn}u_{Rm}$	$\frac{M_{d_{Rn}}}{M_{W_R}}$	$\frac{M_{u_{Rm}}}{M_{W_R}}$	$-\frac{N_c g_R^2}{16\pi^2}\left(-\frac{1}{3}\right)X^*_{mn}X_{mn}\left(\frac{1}{6}-\int_0^1 dz\,\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$e_{Rn}e_{Rn}\nu_{Rm}$	$\frac{M_{e_{Rn}}}{M_{W_R}}$	$\frac{M_{\nu_{Rm}}}{M_{W_R}}$	$-\frac{g_R^2}{16\pi^2}(-1)Y^*_{mn}Y_{mn}\left(\frac{1}{6}-\int_0^1 dz\,\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{D}_R^{++}\tilde{D}_R^{++}\tilde{\chi}_k^+$	$\frac{M_{\tilde{D}_R^{++}}}{M_{W_R}}$	$\frac{M_{\tilde{\chi}_k^+}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}2(\lvert V_{k5}\rvert^2+\lvert U_{k5}\rvert^2)\left(\frac{1}{6}-\int_0^1 dz\,\frac{z^4-z^3+z^2+z^2(\tau^2-1)}{\sigma_k^2(1-z)+z^2+z(\tau^2-1)}\right)$
			$-\frac{g_R^2}{8\pi^2}4\,\mathrm{Re}\,(V^*_{k5}U_{k5})\left(\sigma_k\tau\int_0^1 dz\,\frac{2z^2-z}{\sigma_k^2(1-z)+z^2+z(\tau^2-1)}\right)$
$\tilde{\chi}_k^+\tilde{\chi}_k^+\tilde{D}_R^{++}$	$\frac{M_{\tilde{\chi}_k^+}}{M_{W_R}}$	$\frac{M_{\tilde{D}_R^{++}}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}(\lvert V_{k5}\rvert^2+\lvert U_{k5}\rvert^2)\left(\frac{1}{6}-\int_0^1 dz\,\frac{z^4-z^3+z^2+z^2(\tau_k^2-1)}{\sigma^2(1-z)+z^2+z(\tau_k^2-1)}\right)$
			$-\frac{g_R^2}{8\pi^2}2\,\mathrm{Re}(V^*_{k5}U_{k5})\left(\sigma\tau_k\int_0^1 dz\,\frac{2z^2-z}{\sigma^2(1-z)+z^2+z(\tau_k^2-1)}\right)$
$\tilde{\chi}_j^+\tilde{\chi}_j^+\tilde{\chi}_k^0$	$\frac{M_{\tilde{\chi}_j^+}}{M_{W_R}}$	$\frac{M_{\tilde{\chi}_k^0}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}(\lvert R^L_{kj}\rvert^2+\lvert R^R_{kj}\rvert^2)\left(\frac{1}{6}-\int_0^1 dz\,\frac{z^4-z^3+z^2+z^2(\tau_j^2-1)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}\right)$
			$-\frac{g_R^2}{8\pi^2}2\,\mathrm{Re}\,(R^{L*}_{kj}R^R_{kj})\left(\sigma_k\tau_j\int_0^1 dz\,\frac{2z^2-z}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}\right)$
$H_j^{++}H_j^{--}H_k^+$ $j=1,2;\ k=1,\ldots,6$	$\frac{M_{H_j^{++}}}{M_{W_R}}$	$\frac{M_{H_k^+}}{M_{W_R}}$	$-2\frac{g_R^2}{8\pi^2}a_{1,kj}a_{1,kj}\left(\frac{1}{3}+\int_0^1 dz\,\frac{z^4-2z^3-z^2(\tau_j^2-1)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}\right)$
$H_j^+H_j^-H_k^{++}$ $j=1,\ldots,6;\ k=1,2$	$\frac{M_{H_j^+}}{M_{W_R}}$	$\frac{M_{H_k^{++}}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}a_{1,kj}a_{1,kj}\left(\frac{1}{3}+\int_0^1 dz\,\frac{z^4-2z^3-z^2(\tau_j^2-1)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}\right)$
$H_j^+H_j^-H_k^0$ $j=1,\ldots,6;\ k=1,\ldots,8$	$\frac{M_{H_j^+}}{M_{W_R}}$	$\frac{M_{H_k^0}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}a_{3,kj}a_{3,kj}\left(\frac{1}{3}+\int_0^1 dz\,\frac{z^4-2z^3-z^2(\tau_j^2-1)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}\right)$

TABLE II. (*Continued*).

Particles in the loop	τ	σ	$\kappa_\gamma - 1$
$H_j^+ H_j^- A_k^0$ $j=1,\ldots,6$; $k=1,\ldots,6$	$\frac{M_{H_j^+}}{M_{W_R}}$	$\frac{M_{A_k^0}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2} a_{5,kj} a_{5,kj} \left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 - z^2(\tau_j^2 - 1)}{\sigma_k^2(1-z) + z^2 + z(\tau_j^2 - 1)} \right)$
$W_L W_L H_j^0$ $j=1,\ldots,6$	$\frac{M_{W_L}}{M_{W_R}}$	$\frac{M_{H_j^0}}{M_{W_R}}$	$\frac{g_L^2 g_R^2}{32\pi^2 M_{W_L}^2} a_{8,j} a_{8,j} \left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 + 4z^2}{\sigma_j^2(1-z) + z^2 + z(\tau - 1)} \right)$
$W_R W_R H_j^0$ $j=1,\ldots,6$	1	$\frac{M_{H_j^0}}{M_{W_R}}$	$\frac{g_R^4}{32\pi^2 M_{W_R}^2} a_{7,j} a_{7,j} \left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 + 4z^2}{\sigma_j^2(1-z) + z^2} \right)$
$W_R W_R H_j^{++}$ $j=1,2$	1	$\frac{M_{H_j^{++}}}{M_{W_R}}$	$\frac{g_R^4}{16\pi^2 M_{W_R}^2} a_{9,j} a_{9,j} \left(\frac{1}{3} + \int_0^1 dz \frac{z^4 - 2z^3 + 4z^2}{\sigma_j^2(1-z) + z^2} \right)$

CP and $U(1)_{em}$ invariant vertex is, when all particles are taken on mass shell, $Q^2=0$:

$$\Gamma^{\mu\lambda\rho} = ie\left[AP^{\mu\lambda\rho} + 2(\kappa_\gamma - 1)Q^{\mu\lambda\rho} + 4\frac{\Delta Q_\gamma}{M_W^2} P^\mu q^\lambda q^\rho \right]$$

$$Q^{\mu\lambda\rho} \equiv q^\rho \eta^{\mu\lambda} - q^\lambda \eta^{\mu\rho}, \quad P^{\mu\lambda\rho} \equiv 2p^\mu \eta^{\lambda\rho} + 4Q^{\mu\lambda\rho}. \tag{11}$$

The full magnetic moment is given by $\mu_W = e(1+\kappa_\gamma + \lambda_\gamma)/2M_W$ and the electric quadrupole moment by $-e(\kappa_\gamma - \lambda_\gamma)/M_W^2$. In the standard model $\kappa_\gamma = 1$ and $\lambda_\gamma = 0$. The standard model one loop predictions for the dipole and quadrupole moments of W_L exist in the literature [17]. Contributions to the supersymmetric versions of the SM for the same quantity also exist [18].

The current experimental constraints for the dipole and quadrupole moments are [20,21]

$$-1.3 < \kappa_\gamma < 3.2 \quad \text{for} \quad \lambda_\gamma = 0,$$

$$-0.7 < \lambda_\gamma < 0.7 \quad \text{for} \quad \kappa_\gamma = 1 \tag{12}$$

in $p\overline{p} \to e\nu_e \gamma X$ and $\mu\nu_\mu \gamma X$ at $\sqrt{s} = 1.8$ TeV. In the static limit, constraints on $WW\gamma$ anomalous couplings, as related to the dipole and the quadrupole moment of the W boson, preclude the simultaneous vanishing of both μ_W and Q_W in excess of 95% C.L.

In LRSUSY, at tree level one finds $A=1$, $\kappa_\gamma - 1 = 0$, and $\Delta Q_\gamma = 0$. Deviations from these relations occur at the one-loop level. The basic triangle graphs that must be evaluated can be classified according to the particles in the loop as (1) fermion; (2) scalar; (3) vector; and (4) scalar-vector.

In Tables I–IV, we give, in the last column, a full list of the exact analytical expressions for all the contributions to the magnetic dipole and magnetic quadrupole moments for W_L and W_R in LRSUSY. In the first column we list the particles in the loop, while in the second and third column we give the variables entering the loop integration. The explicit mixing and interaction matrix elements appearing in the tables are given in the Appendix.

IV. MAGNETIC DIPOLE AND QUADRUPOLE MOMENTS: NUMERICAL ANALYSIS

We assume universal boundary conditions for the soft masses at the unification scale $M_{GUT} = 2\times 10^{16}$ GeV. We use the LRSUSY renormalization group equations [19] to evolve all the couplings and the masses from M_{GUT} to M_R and the MSSM renormalization group equations between M_R and M_L. We take as starting parameters $\tan\beta$, A_0, m_0, $M_{1/2}$, and μ. We diagonalize the neutralino and chargino mass matrices and obtain the corresponding masses, imposing present experimental constraints on the lightest neutralino and chargino mass [20]. We proceed in a similar fashion with the slepton and squark mass matrices. We also include physical Higgs contributions by separately diagonalizing the mass matrices for scalars, pseudoscalars, singly and doubly charged scalar particles. All the representative coefficients (mixing matrix elements) are given in the Appendix.

Attempting to scan the parameter space, given the large number of variables, is difficult. To simplify we make some general comments. In the nonsupersymmetric sector, the corrections to the dipole and quadrupole moments are very sensitive to the values of the scalar masses; in the supersymmetric sector, they depend on A_0, m_0, $M_{1/2}$, $\tan\beta$, and μ. We fix the left-right Higgsino mass parameter to M_{LR} so that the mass of the doubly charged Higgsino is $M_{\tilde{D}^{--}} = 200$ GeV. We fix M_R such that $M_{W_R} = 500$ GeV or $M_{W_R} = 1$ TeV and consider both scenarios. We set M_L to give the correct left vector boson masses.

We consider the case where $Q^2=0$, i.e., static moments only, and highlight separately the contributions from the Higgs scalars; squarks and sleptons; and charginos and neutralinos, including doubly charged higgsinos. The contributions from the SM particles to the moments of the W_L boson are well-known and unchanged here. The contributions from the nonsupersymmetric sector to the dipole and quadrupole moments of the W_R boson are given in Table V for $M_{W_R} = 500$ GeV ($M_{W_R} = 1$ TeV). Note in particular the large contribution to the dipole moment from the graph with

TABLE III. Contributions to the magnetic quadrupole moment of W_L to one-loop order.

Particles in the loop	τ	σ	ΔQ_γ
$W_L W_L A$	1	0	$\frac{1}{9}\frac{\alpha}{\pi}$
$W_L W_L Z_L$	1	$\frac{M_{Z_L}}{M_{W_L}}$	$\frac{g_L^2}{16\pi^2}\frac{1}{3}\left(1+\frac{8}{\sigma^2}\right)\int_0^1 dz\,\frac{z^3(1-z)}{\sigma^2(1-z)+z^2}$
$\tilde{u}_{Ln}\tilde{u}_{Ln}\tilde{d}_{Lm}$	$\frac{M_{\tilde{u}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{d}_{Lm}}}{M_{W_L}}$	$-\frac{N_c g_L^2}{16\pi^2}(\frac{2}{3})\tilde{X}^*_{mn}\tilde{X}_{mn}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$\tilde{d}_{Ln}\tilde{d}_{Ln}\tilde{u}_{Lm}$	$\frac{M_{\tilde{d}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{u}_{Lm}}}{M_{W_L}}$	$-\frac{N_c g_L^2}{16\pi^2}(-\frac{1}{3})\tilde{X}^*_{mn}\tilde{X}_{mn}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$\tilde{e}_{Ln}\tilde{e}_{Ln}\tilde{\nu}_{Lm}$	$\frac{M_{\tilde{e}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{\nu}_{Lm}}}{M_{W_L}}$	$-\frac{g_L^2}{16\pi^2}(-1)\tilde{Y}^*_{mn}\tilde{Y}_{mn}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$u_{Ln}u_{Ln}d_{Lm}$	$\frac{M_{u_{Ln}}}{M_{W_L}}$	$\frac{M_{d_{Lm}}}{M_{W_L}}$	$\frac{N_c g_L^2}{16\pi^2}(\frac{2}{3})X^*_{mn}X_{mn}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$d_{Ln}d_{Ln}u_{Lm}$	$\frac{M_{d_{Ln}}}{M_{W_L}}$	$\frac{M_{u_{Lm}}}{M_{W_L}}$	$\frac{N_c g_L^2}{16\pi^2}(-\frac{1}{3})X^*_{mn}X_{mn}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$e_{Ln}e_{Ln}\nu_{Lm}$	$\frac{M_{e_{Ln}}}{M_{W_L}}$	$\frac{M_{\nu_{Lm}}}{M_{W_L}}$	$\frac{g_L^2}{16\pi^2}(-1)Y^*_{mn}Y_{mn}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$\tilde{\chi}_j^+\tilde{\chi}_j^+\tilde{\chi}_k^0$	$\frac{M_{\tilde{\chi}_j^+}}{M_{W_L}}$	$\frac{M_{\tilde{\chi}_k^0}}{M_{W_L}}$	$\frac{g_L^2}{2\pi^2}(\lvert L_{kj}^L\rvert^2+\lvert L_{kj}^R\rvert^2)(\frac{1}{6})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$H_j^{++}H_j^{--}H_k^+$ $j=3,4;\ k=1,\ldots,6$	$\frac{M_{H_j^{++}}}{M_{W_L}}$	$\frac{M_{H_k^+}}{M_{W_L}}$	$-2\frac{g_L^2}{8\pi^2}a_{2,kj}a_{2,kj}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$H_j^+H_j^-H_k^{++}$ $j=1,\ldots,6;\ k=3,4$	$\frac{M_{H_j^+}}{M_{W_L}}$	$\frac{M_{H_k^{++}}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{2,kj}a_{2,kj}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$H_j^+H_j^-H_k^0$ $j=1,\ldots,6;\ k=1,\ldots,8$	$\frac{M_{H_j^+}}{M_{W_L}}$	$\frac{M_{H_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{4,kj}a_{4,kj}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$H_j^+H_j^-A_k^0$ $j=1,\ldots,6;\ k=1,\ldots,6$	$\frac{M_{H_j^+}}{M_{W_L}}$	$\frac{M_{A_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{6,kj}a_{6,kj}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$W_L W_L H_j^0$ $j=1,\ldots,6$	1	$\frac{M_{H_j^0}}{M_{W_L}}$	$\frac{g_L^4}{32\pi^2 M_{W_L}^2}a_{8,j}a_{8,j}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2}$
$W_R W_R H_j^0$ $j=1,\ldots,6$	$\frac{M_{W_R}}{M_{W_L}}$	$\frac{M_{H_j^0}}{M_{W_L}}$	$\frac{g_L^2 g_R^2}{32\pi^2 M_{W_L}^2}a_{8,j}a_{8,j}(\frac{2}{3})\int_0^1 dz\,\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2+z(\tau^2-1)}$

W_R, W_R, and Z_R in the loop. The contribution to both the dipole and quadrupole moments of W_R is in that case dependent only on the mass ratio of Z_R and W_R, and thus on the ratio of g_L and g_R. So, although this appears to be cancelled in the $M_{W_R}=500$ GeV scenario from contributions coming from quarks, the later contributions are W_R mass-dependent, while the former are not. This is a characteristic feature of the dipole moments in the right-handed gauge sector of the model.

We now outline the main features of the contributions from the supersymmetric and Higgs sector of the LRSUSY model to the dipole and quadrupole moments of the W_R and W_L.

A. Higgs boson contribution

In the MSSM, it was found that the bulk of the Higgs bosons contribution to Δk_γ, ΔQ_γ of W_L is saturated by the contributions from the lightest *CP*-even Higgs boson, whose one-loop corrected mass does not exceed ≈ 140 GeV. The

TABLE IV. Contributions to the magnetic quadrupole moment of W_R to one-loop order.

Particles in the loop	τ	σ	ΔQ_γ
$W_R W_R A$	1	0	$\frac{1}{9}\frac{\alpha}{\pi}$
$W_R W_R Z_L$	1	$\frac{M_{Z_L}}{M_{W_R}}$	$\frac{e^2\tan^2\theta_W}{16\pi^2}\frac{8+\sigma^2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma^2(1-z)+z^2}$
$W_R W_R Z_R$	1	$\frac{M_{Z_R}}{M_{W_R}}$	$\frac{g_R^2}{16\pi^2}\frac{2}{3}\left(1+\frac{8}{\sigma^2}\right)\int_0^1 dz\frac{z^3(1-z)}{\sigma^2(1-z)+z^2}$
$\tilde{u}_{Rn}\tilde{u}_{Rn}\tilde{d}_{Rm}$	$\frac{M_{\tilde{u}_{Rn}}}{M_{W_R}}$	$\frac{M_{\tilde{d}_{Rm}}}{M_{W_R}}$	$-\frac{N_c g_R^2}{16\pi^2}(\frac{2}{3})\tilde{X}^*_{mn}\tilde{X}_{mn}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$\tilde{d}_{Rn}\tilde{d}_{Rn}\tilde{u}_{Rm}$	$\frac{M_{\tilde{d}_{Rn}}}{M_{W_R}}$	$\frac{M_{\tilde{u}_{Rm}}}{M_{W_R}}$	$-\frac{N_c g_R^2}{16\pi^2}(-\frac{1}{3})\tilde{X}^*_{mn}\tilde{X}_{mn}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$\tilde{e}_{Rn}\tilde{e}_{Rn}\tilde{\nu}_{Rm}$	$\frac{M_{\tilde{e}_{Rn}}}{M_{W_R}}$	$\frac{M_{\tilde{\nu}_{Rm}}}{M_{W_R}}$	$-\frac{g_L^2}{16\pi^2}(-1)\tilde{Y}^*_{mn}\tilde{Y}_{mn}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$u_{Rn}u_{Rn}d_{Rm}$	$\frac{M_{u_{Rn}}}{M_{W_R}}$	$\frac{M_{d_{Rm}}}{M_{W_R}}$	$\frac{N_c g_R^2}{16\pi^2}(\frac{2}{3})X^*_{mn}X_{mn}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$d_{Rn}d_{Rn}u_{Rm}$	$\frac{M_{d_{Rn}}}{M_{W_R}}$	$\frac{M_{u_{Rm}}}{M_{W_R}}$	$\frac{N_c g_R^2}{16\pi^2}(-\frac{1}{3})X^*_{mn}X_{mn}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$e_{Rn}e_{Rn}\nu_{Rm}$	$\frac{M_{e_{Rn}}}{M_{W_R}}$	$\frac{M_{\nu_{Rm}}}{M_{W_R}}$	$\frac{g_R^2}{16\pi^2}(-1)Y^*_{mn}Y_{mn}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}$
$\tilde{D}_R^{++}\tilde{D}_R^{++}\tilde{\chi}_k^+$	$\frac{M_{\tilde{D}_R^{++}}}{M_{W_R}}$	$\frac{M_{\tilde{\chi}_k^+}}{M_{W_R}}$	$\frac{g_R^2}{2\pi^2}4\,\mathrm{Re}(V^*_{k5}U_{k5})(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau^2-1)}$
$\tilde{\chi}_k^+\tilde{\chi}_k^+\tilde{D}_R^{++}$	$\frac{M_{\tilde{\chi}_k^+}}{M_{W_R}}$	$\frac{M_{\tilde{D}_R^{++}}}{M_{W_R}}$	$\frac{g_R^2}{2\pi^2}(\lvert V_{k5}\rvert^2+\lvert U_{k5}\rvert^2)(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau^2-1)}$
$\tilde{\chi}_j^+\tilde{\chi}_j^+\tilde{\chi}_k^0$	$\frac{M_{\tilde{\chi}_j^+}}{M_{W_R}}$	$\frac{M_{\tilde{\chi}_k^0}}{M_{W_R}}$	$\frac{g_R^2}{2\pi^2}(\lvert R^L_{kj}\rvert^2+\lvert R^R_{kj}\rvert^2)(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$H_j^{++}H_j^{--}H_k^+$ $j=1,2;\ k=1,\ldots,6$	$\frac{M_{H_j^{++}}}{M_{W_R}}$	$\frac{M_{H_k^+}}{M_{W_R}}$	$-2\frac{g_R^2}{8\pi^2}a_{1,kj}a_{1,kj}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$H_j^+H_j^-H_k^{++}$ $j=1,\ldots,6;\ k=1,2$	$\frac{M_{H_j^+}}{M_{W_R}}$	$\frac{M_{H_k^{++}}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}a_{1,kj}a_{1,kj}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$H_j^+H_j^-H_k^0$ $j=1,\ldots,6;\ k=1,\ldots,8$	$\frac{M_{H_j^+}}{M_{W_R}}$	$\frac{M_{H_k^0}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}a_{3,kj}a_{3,kj}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$H_j^+H_j^-A_k^0$ $j=1,\ldots,6;\ k=1,\ldots,6$	$\frac{M_{H_j^+}}{M_{W_R}}$	$\frac{M_{A_k^0}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}a_{5,kj}a_{5,kj}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z(\tau_j^2-1)}$
$W_LW_LH_j^0$ $j=1,\ldots,6$	$\frac{M_{W_L}}{M_{W_R}}$	$\frac{M_{H_j^0}}{M_{W_R}}$	$\frac{g_L^2g_R^2}{32\pi^2M_{W_L}^2}a_{8,j}a_{8,j}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2+z(\tau^2-1)}$
$W_RW_RH_j^0$ $j=1,\ldots,6$	1	$\frac{M_{H_j^0}}{M_{W_R}}$	$\frac{g_R^4}{32\pi^2M_{W_R}^2}a_{7,j}a_{7,j}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2}$
$W_RW_RH_j^{++}$ $j=1,2$	1	$\frac{M_{H_j^{++}}}{M_{W_R}}$	$\frac{g_R^4}{16\pi^2M_{W_R}^2}a_{9,j}a_{9,j}(\frac{2}{3})\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2}$

TABLE V. Contributions to the dipole and quadrupole moments of W_R in units of $g_R^2/16\pi^2$ for $M_{W_R}=500$ GeV (1 TeV).

Particles in the loop	Δk_γ	ΔQ_γ
$W_R W_R A$	3.9×10^{-3}	2.6×10^{-4}
$W_R W_R Z_L$	0.2(0.23)	0.015(0.017)
$W_R W_R Z_R$	−2.39	0.09
t,b	−0.82(0.47)	0.38(−0.63)
c,s	2.53(−0.21)	0.41(−0.05)
u,d	0.17(0.17)	−0.22(−0.22)
e,ν_e	−0.16(−0.16)	0.22(0.22)
μ,ν_μ	−0.16(−0.16)	0.22(0.22)
τ,ν_τ	−0.14(−0.15)	0.23(0.23)

situation is reproduced in LRSUSY, for a region of the parameter space, where 125 GeV$\approx M_{H^0}\leqslant M_{H^{++}}$. The contribution from the doubly charged Higgs bosons is at most 20% of the lightest Higgs boson contribution, and that for very light doubly charged bosons. The total Higgs contribution could reach values of (0.05,−0.1) for the dipole moment, while the Higgs contribution is smaller for the quadrupole moment (0.02,−0.04). For the W_R dipole moments the Higgs contribution is of the same order of magnitude (0.1, −0.2), while for the quadrupole moment it varies between (0.05 and −0.35) for different W_R masses. We show the results in Fig. 1(a) for dipole moments, and in Fig. 1(b) for quadrupole moments. Note that we cite moments of W_L in units of $g_L^2/16\pi^2$ and of W_R in units of $g_R^2/16\pi^2$, so both can be read from the figures, and compared for $g_L=g_R$.

B. Squarks and slepton contribution

We have found the total contribution from squarks and sleptons to the dipole and quadrupole of the W_L boson to be negligible compared to the contribution from other sectors, i.e., of order 10^{-3} or smaller (in units of $g_L^2/16\pi^2$) for the magnetic dipole moment; and between (−0.01 and −0.005) for the magnetic quadrupole moment. This agrees roughly with the MSSM estimate. The contribution of squarks and sleptons to the dipole moment of W_R is larger, varying between (−0.04 and −0.1), but still smaller comparatively to other contributions; while the quadrupole moments vary between (−0.04 and −0.01). We show the results in Fig. 2(a) for dipole moments, and Fig. 2(b) for quadrupole moments.

C. Charginos and neutralinos contribution

The contribution from charginos and neutralinos in MSSM was found to be, under certain conditions, the dominant supersymmetric contribution, and for cases in which $M_{1/2}\leqslant A_0, m_0$ could even reach the SM limits. LRSUSY has 5 charginos and 11 neutralinos. The contribution to the magnetic dipole moment of the W_L is large and depends on the $sign(\mu)$. It is always negative and could reach between (−0.64 and −0.7) for $sign(\mu)>0$; and between (−0.6 and 0) for $sign(\mu)<0$, all in units of $g_L^2/16\pi^2$. For the quadrupole moment, this is still the dominant supersymmetric contribution, however, there it is of order 10^{-2} only. There have been indications that the new accurate measurements of the anomalous magnetic moment of the muon restrict the μ parameter to be positive, as does SO(10) unification, while the decay $b\to s\gamma$ favors a negative sign. A detailed analysis of the $B_s\to X_s\gamma$ in LRSUSY [22] shows that for light squark-gaugino mass scenario, and low $\tan\beta$, the bound on $b\to s\gamma$ is satisfied for either sign of the μ parameter. However, when μ increases beyond 325–350 GeV, the branching ratio for $B_s\to X_s\gamma$ exceeds the experimental value. The parameter space is less restrictive for $sign(\mu)<0$. The right-

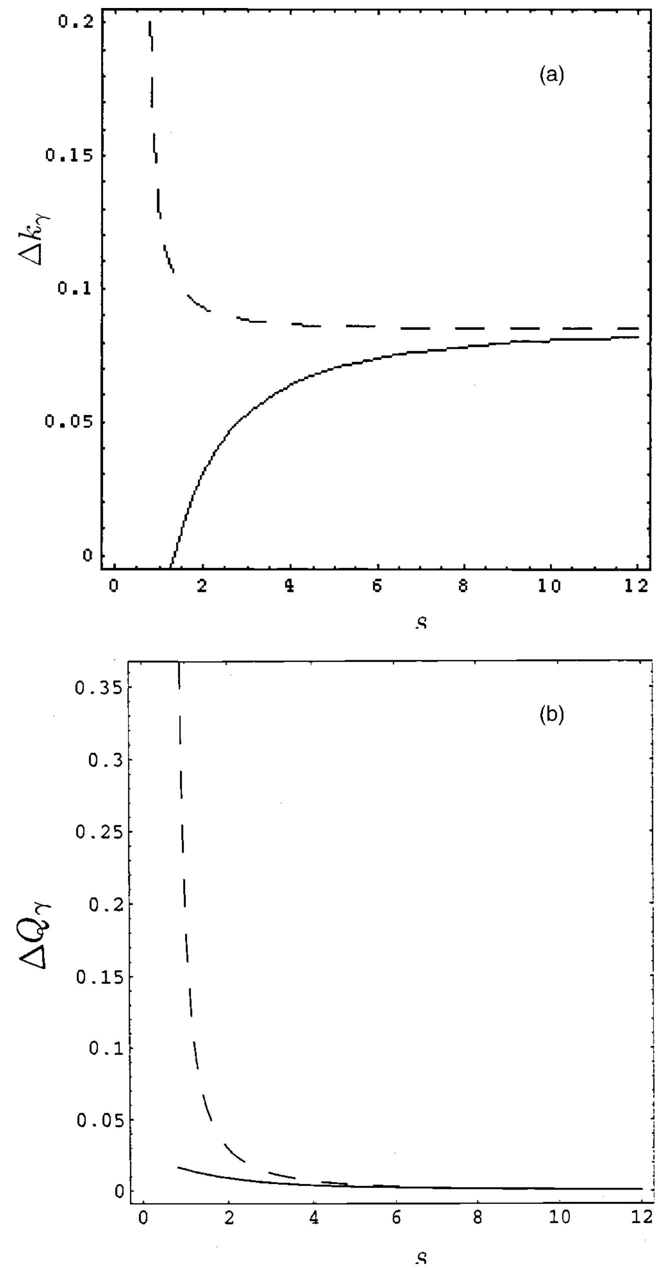

FIG. 1. Contribution of the Higgs bosons to the (a) dipole magnetic moment and (b) quadrupole moment of the W_L, W_R bosons as a function of the lightest Higgs boson mass. The solid graph corresponds to W_L moments, the dashed to W_R moments. Here $s=M_{H^0}/M_{W_{L,R}}$.

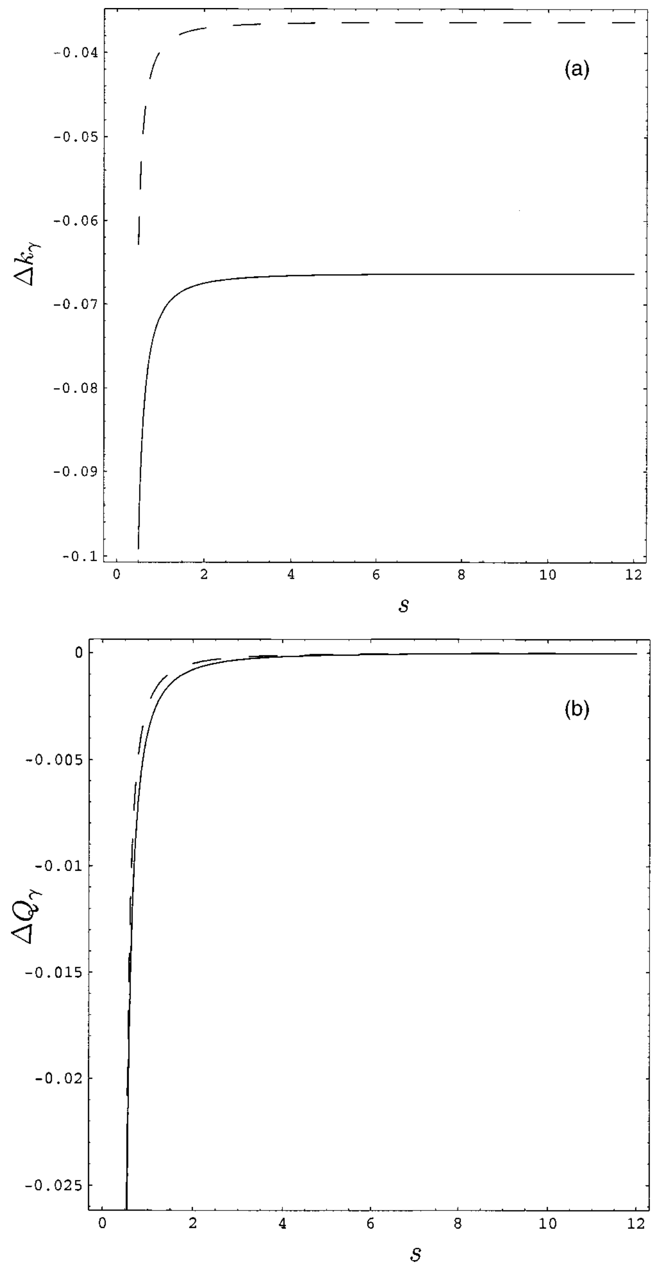

FIG. 2. Contribution of the squarks and sleptons to the (a) dipole magnetic moment and (b) quadrupole moment of the W_L, W_R bosons as function of the universal scalar mass m_0. We take $s = m_0/M_{WL,R}$. The solid graph corresponds to $\mu = 100$ GeV, the dashed to $\mu = 200$ GeV.

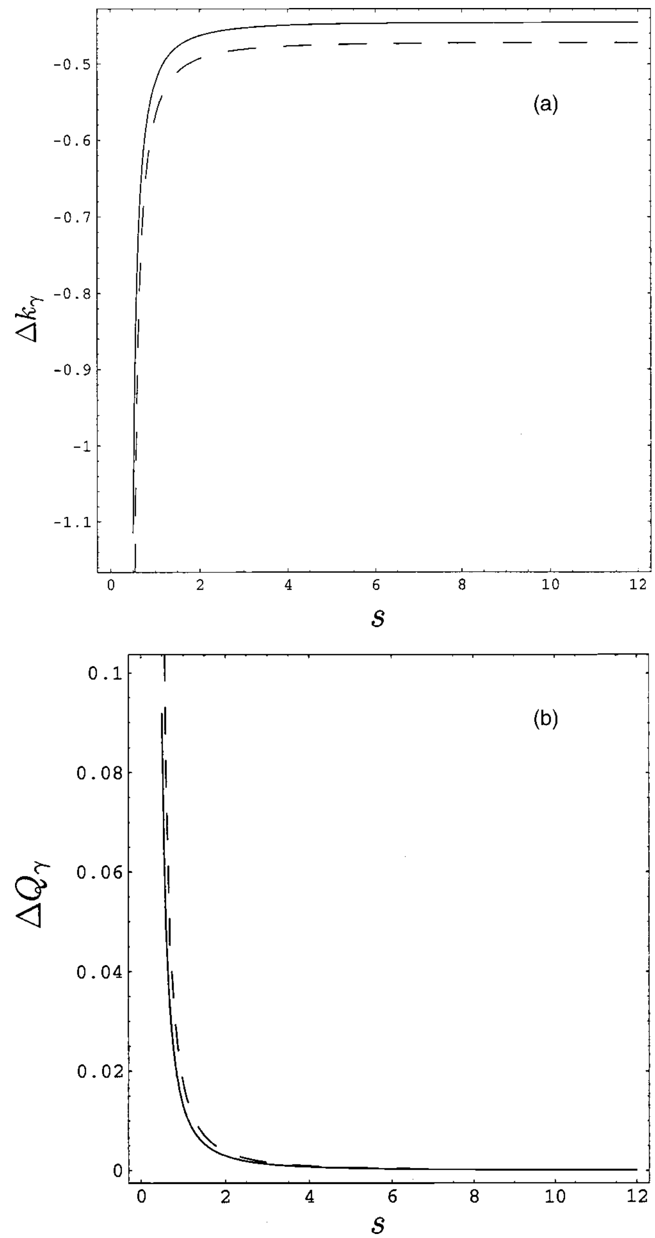

FIG. 3. Contribution of the charginos and neutralinos to the (a) dipole magnetic moment and (b) quadrupole moment of the W_L, W_R bosons as a function of the lightest neutralino mass. Here $s = m_{\chi^0}/M_{WL,R}$. We take the $SU(2)_L$ gaugino mass $M_{\tilde{g}_L} = 200$ GeV and $SU(2)_R$ gaugino mass $M_{\tilde{g}_R} = 1$ TeV. The solid graph corresponds to $sign(\mu) > 0$, the dashed one to $sign(\mu) < 0$.

handed gaugino mass (which may be heavy or light) does not influence the results directly, only through physical chargino and neutralino masses. If $M_{\tilde{g}_R}$ becomes very large, the right-handed gaugino effectively decouples from the light chargino spectrum, and the restrictions on μ depend only on the $M_{\tilde{g}_L}$, $\tan\beta$, and m_0 parameters. This occurs for $M_{\tilde{g}_R} \geqslant 100 M_{\tilde{g}_L}$ and it has no discernable effect on the restrictions on the μ parameter.

A comment is in order about the doubly charged higgsino contribution. Although we have taken the mass of the doubly charged higgsino to be light (about 200 GeV), the corresponding contribution depends on the mixing matrix element of the corresponding singly charged higgsino, which is small. So the contribution from the doubly charged higgsino is much smaller than other chargino-neutralino contributions, and thus the static moments of W bosons are not sensitive to variations in the parameter A_{LR}. The chargino and neutralino contribution is the largest supersymmetric contribution to the dipole and quadrupole moments of W_R reaching $(-0.6, -1.6)$ for dipole moments, and of order of 10^{-1} for quadrupole moments. We show the results in Fig. 3(a) for dipole moments, and Fig. 3(b) for quadrupole moments.

For all the above considerations, we have taken $\tan\beta = 10$. Varying $\tan\beta$ has little impact of the static moments for low-medium values of $\tan\beta$, and is indistinguishable from choosing a different μ parameter at high values. As such, the dipole and quadrupole moments are not a sensitive probe of $\tan\beta$. They are somewhat sensitive to values of m_0, A_0, but only somewhat, because these parameters mostly affect the scalar fermion sector. The static moments are more sensitive to parameters in the gaugino-higgsino sectors, such as gaugino masses $M_{1/2}$ and the parameter μ. We have chosen to highlight the dependence on chargino, neutralino, and slepton-squark masses, since once more information on the mass spectra is available, we would be able to rule out some regions of the parmeter space.

V. CONCLUSION

We have presented a complete analytical and numerical study of the static *CP*-conserving moments of W_L and W_R in the context of a fully supersymmetric left-right model. The only model-dependent choice in this model is the choice of triplets versus doublets Higgs to break $SU(2)_R$, and we justified this Higgs representation as supporting of the seesaw mechanism for neutrino masses. We first gave complete analytical expressions for the dipole and quadrupole moments as functions of mass ratios of the particles in the model. For W_L boson, we analyzed separately the contributions from the Higgs bosons, squarks-sleptons, and chargino-neutralino graphs. We find that the Higgs contribution is similar to the MSSM one and, for a wide range of scalar mass parameters, it is saturated by the contribution from the lightest scalar Higgs boson. The contribution from the doubly charged Higgs boson can account for at most 20% of the total Higgs contribution, and that only for very light doubly charged scalars. For all of the supersymmetric parameter space scanned, the slepton-squark contribution is the smallest and negligible; and the chargino-neutralino contribution the largest, although here there are significant differences between MSSM and LRSUSY. For example, for the lightest chargino and neutralino $m_{\chi^{0,\pm}} \leqslant 80$ GeV, this contribution is always less than of order 1, regardless of the $sign(\mu)$.

For W_R, the largest potential contribution to the dipole moment comes from the $W_R W_R Z_R$ graph: it is $\approx -2.4(g_R^2/16\pi^2)$, it is independent of the masses of W_R and Z_R bosons, and depends only on the M_{Z_R}/M_{W_R} mass ratio. This contribution may or may not be cancelled by quark contributions, or contributions coming from the supersymmetric sector, which, however, are dependent on other mass parameters. The Higgs boson contribution can be at most of $\mathcal{O}(10^{-1})$. In the supersymmetric sector, the contribution from squarks and sleptons is small; however, the contribution from charginos and neutralinos is potentially large and becomes of $\mathcal{O}(1)$ for $m_{\chi^0}/M_{W_R} < 0.5$. Although we have chosen to show the results for a relatively light W_R, the extension to a heavy W_R is straightforward.

The outcome of the present analysis is that the dipole and quadrupole moments of the W boson(s), as a signal for new physics, are model dependent. At this stage, further knowledge about the supersymmetric masses is needed to exclude regions of the parameters space. However, in view of the projected sensitivities of the Next Linear Collider (NLC), precise measurements of the static moments of the $W_{L,R}$ bosons would provide information on the gauge symmetry of the theory at high energies, complementary to experimental tests of the fermionic sectors.

ACKNOWLEDGMENT

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APPENDIX

In this appendix we give the expressions for the interaction terms and mixing matrices which appear in the text.

1. Chargino mixing

The relevant Feynman rules used in the calculation are listed in this appendix. The terms relevant to the masses of charginos in the Lagrangian are

$$\mathcal{L}_C = -\frac{1}{2}(\psi^{+T}, \psi^{-T}) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{H.c.}, \tag{A1}$$

where $\psi^{+T} = (-i\lambda_L^+, -i\lambda_R^+, \tilde{\phi}_{1u}^+, \tilde{\phi}_{1d}^+, \tilde{\Delta}_R^+)^T$ and $\psi^{-T} = (-i\lambda_L^-, -i\lambda_R^-, \tilde{\phi}_{2u}^-, \tilde{\phi}_{2d}^-, \tilde{\delta}_R^-)^T$, and

$$X = \begin{pmatrix} M_{\tilde{g}_L} & 0 & \frac{g_L\kappa_d}{\sqrt{2}} & 0 & 0 \\ 0 & M_{\tilde{g}_R} & \frac{g_R\kappa_d}{\sqrt{2}} & 0 & g_R v_{\Delta_R} \\ \frac{g_L\kappa_u}{\sqrt{2}} & \frac{g_R\kappa_u}{\sqrt{2}} & 0 & -\mu_1 & 0 \\ 0 & 0 & -\mu_1 & 0 & 0 \\ 0 & g_R v_{\delta_R} & 0 & 0 & -\mu_3 \end{pmatrix}, \tag{A2}$$

where we have taken, for simplification, $\mu_{ij} = \mu$. The chargino mass eigenstates χ_i are obtained by

$$\chi_i^+ = V_{ij}\psi_j^+, \quad \chi_i^- = U_{ij}\psi_j^-, \quad i,j = 1, \ldots 5, \tag{A3}$$

with V and U unitary matrices satisfying

$$U^* X V^{-1} = X_D. \tag{A4}$$

The diagonalizing matrices U^* and V are obtained by computing the eigenvectors corresponding to the eigenvalues of $X^\dagger X$ and $XX^\dagger$, respectively.

2. Neutralino mixing

The terms relevant to the masses of neutralinos in the Lagrangian are

$$\mathcal{L}_N = -\frac{1}{2}\psi^{0T} Y \psi^0 + \text{H.c.}, \tag{A5}$$

where $\psi^0 = (-i\lambda_L^0, -i\lambda_R^0, -i\lambda_V, \tilde{\phi}_{1u}^0, \tilde{\phi}_{2d}^0, \tilde{\Delta}_R^0, \tilde{\delta}_R^0, \tilde{\phi}_{1d}^0, \tilde{\phi}_{2u}^0)^T$, and

$$Z=\begin{pmatrix} M_{\tilde{g}_L} & 0 & 0 & -\frac{g_L\kappa_u}{2} & \frac{g_L\kappa_d}{2} & 0 & 0 & 0 & 0 \\ 0 & M_{\tilde{g}_R} & 0 & \frac{g_L\kappa_u}{2} & \frac{g_L\kappa_d}{2} & -g_R v_{\Delta_R} & -g_R v_{\delta_R} & 0 & 0 \\ 0 & 0 & M_{\tilde{g}_V} & 0 & 0 & 2g_V v_{\Delta_R} & 2g_V v_{\delta_R} & 0 & 0 \\ -\frac{g_L\kappa_u}{2} & \frac{g_R\kappa_u}{2} & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 \\ \frac{g_L\kappa_d}{2} & -\frac{g_R\kappa_d}{2} & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -g_R v_{\Delta_R} & 2g_V v_{\Delta_R} & 0 & 0 & 0 & -\mu_3 & 0 & 0 \\ 0 & -g_R v_{\delta_R} & 2g_V v_{\delta_R} & 0 & 0 & -\mu_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0 \end{pmatrix}. \tag{A6}$$

The mass eigenstates are defined by

$$\chi_i^0 = M_{ij}\psi_j^0 \quad (i,j=1,2,\ldots,9), \tag{A7}$$

where M is a unitary matrix chosen such that

$$MZM^T = Z_D, \tag{A8}$$

and Z_D is a diagonal matrix with non-negative entries.

3. Vector boson-fermion interactions

The interaction of fermions with vector bosons arise from the fermion kinetic energy term in the Lagrangian density. Rewriting the complete interaction with physical quarks, leptons, charginos, and neutralinos, one must introduce two CKM matrices X_{mn} in the quark sector, and Y_{mn} in the lepton sector. In addition we introduce the physical charginos and neutralinos previously defined in this appendix. The relevant Lagrangian density is given by

$$\begin{aligned}\mathcal{L} = {} & \frac{g_L}{\sqrt{2}} W_\mu^{L+}(X_{mn}\overline{u}_m\gamma^\mu\gamma_L d_n + Y_{mn}\overline{\nu}_m\gamma^\mu\gamma_L e_n) + \text{H.c.} \\ & + \frac{g_R}{\sqrt{2}} W_\mu^{R+}(X^*_{mn}\overline{u}_m\gamma^\mu\gamma_R d_n + Y^*_{mn}\overline{\nu}_m\gamma^\mu\gamma_R e_n) + \text{H.c.} \\ & + g_L W_\mu^{L+}(-\bar{\tilde{D}}_L^{++}\gamma^\mu\tilde{D}_L^{+} + \bar{\tilde{D}}_L^{+}\gamma^\mu\tilde{D}_L^{0}) \\ & - g_R W_\mu^{R+}\bar{\tilde{D}}_R^{++}\gamma^\mu(V^*_{k5}\gamma_L + U_{k5}\gamma_R)\tilde{\chi}_k^{+} + \text{H.c.} \\ & + g_L W_\mu^{L+}\bar{\tilde{\chi}}_j^{+}\gamma^\mu(L_{jk}^L\gamma_L + L_{jk}^R\gamma_R)\tilde{\chi}_k^0 \\ & + g_R W_\mu^{R+}\chi_j^{+}\gamma^\mu(R_{jk}^L\gamma_L + R_{jk}^R\gamma_R)\tilde{\chi}_k^0 + \text{H.c.},\end{aligned} \tag{A9}$$

where the matrix elements L_{jk}, R_{jk} which appear in the expressions for the static quantities of the $W_{L,R}$ bosons are given by

$$\begin{aligned} L_{jk}^L &= -M^*_{k1}V_{j1} + \frac{1}{\sqrt{2}}M^*_{k5}V_{j4} + \frac{1}{\sqrt{2}}M^*_{k9}V_{j3}, \\ L_{jk}^R &= -U^*_{j1}M_{k1} - \frac{1}{\sqrt{2}}U^*_{j3}M_{k4} - \frac{1}{\sqrt{2}}U^*_{j4}M_{k8}, \\ R_{jk}^L &= -M^*_{k2}V_{j2} + \frac{1}{\sqrt{2}}M^*_{k5}V_{j4} + M^*_{k6}V_{j5} + \frac{1}{\sqrt{2}}M^*_{k9}V_{j3}, \\ R_{jk}^R &= -U^*_{j2}M_{k2} - \frac{1}{\sqrt{2}}U^*_{j3}M_{k4} + U^*_{j5}M_{k7} + \frac{1}{\sqrt{2}}U^*_{j4}M_{k8}. \end{aligned} \tag{A10}$$

4. Scalar mixing

Here we give expressions for the matrix elements which appear in the interaction between vector bosons and scalars. The interactions between vector bosons and scalars arise from the kinetic energy term for the gauge bosons in the Lagrangian density. We denote by $x_{L,R}$ the Higgs fields before mixing, and by $y_{L,R}$ the Higgs fields after mixing. The Higgs scalar fields are defined as

Doubly Charged Fields

$$\begin{aligned} & x_R^{++T} \equiv (\Delta_R^{++}, \delta_R^{--*}), \quad x_R^{--T} \equiv (\Delta_R^{++*}, \delta_R^{--*}), \\ & y_R^{\pm\pm T} \equiv (H_1^{\pm\pm}, H_2^{\pm\pm}), \\ & x_L^{++T} \equiv (\Delta_L^{++}, \delta_L^{--*}), \quad x_L^{--T} \equiv (\Delta_L^{++*}, \delta_L^{--*}), \\ & y_L^{\pm\pm T} \equiv (H_3^{\pm\pm}, H_4^{\pm\pm}). \end{aligned} \tag{A11}$$

Singly Charged Fields

$$x^{+T}\equiv(\Delta_L^+,\delta_L^{-*},\phi_{2d}^{-*},\phi_{1u}^+,\phi_{2u}^{-*},\phi_{1d}^+,\Delta_R^+,\delta_R^{-*}),$$
$$x^{-T}\equiv(\Delta_L^{+*},\delta_L^-,\phi_{2d}^-,\phi_{1u}^{+*},\phi_{2u}^-,\phi_{1d}^{+*},\Delta_R^{+*},\delta_R^-), \tag{A12}$$
$$y^{\pm T}\equiv(H_1^\pm,H_2^\pm,H_3^\pm,H_4^\pm,H_5^\pm,H_6^\pm,G_1^\pm,G_2^\pm).$$

Neutral Fields

$$x_s^{0T}\equiv(H_{\Delta_L},H_{\delta_L},H_{1d}^0,H_{2u}^0,H_{1u}^0,H_{2d}^0,H_{\Delta_R},H_{\delta_R}),$$
$$y_s^{0T}\equiv(H_1^0,H_2^0,H_3^0,H_4^0,H_5^0,H_6^0,H_7^0,H_8^0),$$
$$x_p^{0T}\equiv(z_{\Delta_L},z_{\delta_L},z_{1d}^0,z_{2u}^0,z_{1u}^0,z_{2d}^0,z_{\Delta_R},z_{\delta_R}), \tag{A13}$$
$$y_p^{0T}\equiv(A_1^0,A_2^0,A_3^0,A_4^0,A_5^0,A_6^0,G_1^0,G_2^0),$$

where the indices "s" and "p" stand for scalar and pseudoscalar, respectively. There are two charged Goldstone bosons for the left-handed and the right-handed charged vector bosons, and two neutral Goldstone bosons for the Z_L and Z_R bosons. They have zero mass. We define them to be the seventh and eighth component of $H^\pm$ and A^0, in order to simplify use of the summation convention. The mass matrices M are real and symmetric and diagonalized by orthogonal matrices R defined as

$$(R_R^{\pm\pm})_{ij}(M_R^{\pm\pm})_{jk}(R_R^{\pm\pm})_{lk}=\mathrm{diag}(m_1^{\pm\pm},m_2^{\pm\pm}),$$
$$(R_L^{\pm\pm})_{ij}(M_L^{\pm\pm})_{jk}(R_L^{\pm\pm})_{lk}=\mathrm{diag}(m_3^{\pm\pm},m_4^{\pm\pm}),$$
$$(R^\pm)_{ij}(M^\pm)_{jk}(R^\pm)_{lk}=\mathrm{diag}(m_1^\pm,\ldots,m_6^\pm,0,0), \tag{A14}$$
$$(R_s^0)_{ij}(M_s^0)_{jk}(R_s^0)_{lk}=\mathrm{diag}(m_{s1}^0,\ldots,m_{s8}^0),$$
$$(R_p^0)_{ij}(M_p^0)_{jk}(R_p^0)_{lk}=\mathrm{diag}(m_{p1}^0,\ldots,m_{s6}^0,0,0),$$

where m_{s1} is the mass of the lightest Higgs scalar. Rewriting the Higgs part of the potential in terms of physical states (i.e., diagonalizing the scalar mass matrices), we introduce new fields by

$$y_{Ri}^{\pm\pm}=(R_R^{\pm\pm})_{ij}x_{Rj}^{\pm\pm}, \quad y_{Li}^{\pm\pm}=(R_L^{\pm\pm})_{ij}x_{Lj}^{\pm\pm},$$
$$y_i^\pm=(R^\pm)_{ij}x_j^\pm,$$
$$y_{si}^0=(R_s^0)_{ij}x_{sj}^0, \quad y_{pi}^0=(R_p^0)_{ij}x_{pj}^0. \tag{A15}$$

Then the coefficients that appear in Tables I–IV can be written in terms of the matrices which diagonalize the scalar mass matrices as:

$$a_{1,jk}=(R_R^{\pm\pm})_{j1}(R^\pm)_{k7}+(R_R^{\pm\pm})_{j2}(R^\pm)_{k8},$$
$$a_{2,jk}=(R_L^{\pm\pm})_{j1}(R^\pm)_{k1}+(R_L^{\pm\pm})_{j2}(R^\pm)_{k2},$$
$$a_{3,jk}=\frac{1}{2}(R^\pm)_{j3}(R_s^0)_{k3}-\frac{1}{2}(R^\pm)_{j4}(R_s^0)_{k4}+\frac{1}{2}(R^\pm)_{j5}(R_s^0)_{k5}-\frac{1}{2}(R^\pm)_{j6}(R_s^0)_{k6}-\frac{1}{\sqrt{2}}(R^\pm)_{j7}(R_s^0)_{k7}-\frac{1}{\sqrt{2}}(R^\pm)_{j8}(R_s^0)_{k8},$$
$$a_{4,jk}=\frac{1}{\sqrt{2}}(R^\pm)_{j1}(R_s^0)_{k1}-\frac{1}{\sqrt{2}}(R^\pm)_{j2}(R_s^0)_{k2}+\frac{1}{2}(R^\pm)_{j3}(R_s^0)_{k3}-\frac{1}{2}(R^\pm)_{j4}(R_s^0)_{k4}+\frac{1}{2}(R^\pm)_{j5}(R_s^0)_{k5}-\frac{1}{2}(R^\pm)_{j6}(R_s^0)_{k6},$$
$$a_{5,jk}=\frac{1}{2}(R^\pm)_{j3}(R_p^0)_{k3}+\frac{1}{2}(R^\pm)_{j4}(R_p^0)_{k4}+\frac{1}{2}(R^\pm)_{j5}(R_p^0)_{k5}+\frac{1}{2}(R^\pm)_{j6}(R_p^0)_{k6}+\frac{1}{\sqrt{2}}(R^\pm)_{j7}(R_p^0)_{k7}-\frac{1}{\sqrt{2}}(R^\pm)_{j8}(R_p^0)_{k8},$$
$$a_{6,jk}=\frac{1}{\sqrt{2}}(R^\pm)_{j1}(R_p^0)_{k1}-\frac{1}{\sqrt{2}}(R^\pm)_{j2}(R_p^0)_{k2}+\frac{1}{2}(R^\pm)_{j3}(R_p^0)_{k3}+\frac{1}{2}(R^\pm)_{j4}(R_p^0)_{k4}+\frac{1}{2}(R^\pm)_{j5}(R_p^0)_{k5}+\frac{1}{2}(R^\pm)_{j6}(R_p^0)_{k6},$$
$$a_{7,j}=\frac{\kappa_u}{2}(R_s^0)_{j5}+\frac{\kappa_d}{2}(R_s^0)_{j6}+v_{\Delta_R}(R_s^0)_{j7}+v_{\delta_R}(R_s^0)_{j8},$$
$$a_{8,j}=\frac{\kappa_u}{2}(R_s^0)_{j5}+\frac{\kappa_d}{2}(R_s^0)_{j6}, \tag{A16}$$
$$a_{9,j}=v_{\Delta_R}(R_R^{\pm\pm})_{j1}+v_{\delta_R}(R_R^{\pm\pm})_{j2},$$
$$a_{10,j}=-\frac{\kappa_u}{2}(R^\pm)_{j5}+\frac{\kappa_d}{2}(R^\pm)_{j6}+\frac{v_{\Delta_R}}{\sqrt{2}}(R^\pm)_{j7}+\frac{v_{\delta_R}}{\sqrt{2}}(R^\pm)_{j8},$$
$$a_{11,j}=-\frac{\kappa_u}{2}(R^\pm)_{j5}+\frac{\kappa_d}{2}(R^\pm)_{j6},$$

$$a_{12,j}=-\frac{1+\sin^2\phi}{\sqrt{2}\cos\phi}[v_{\Delta_R}(R^{\pm})_{j7}+v_{\delta_R}(R^{\pm})_{j8}],$$

$$a_{13,j}=\cos\theta_W\left[-\frac{\kappa_u}{2}(R^{\pm})_{j5}+\frac{\kappa_d}{2}(R^{\pm})_{j6}\right]$$
$$+\frac{\sin^2\theta_W}{\cos\theta_W}\left[-\frac{v_{\Delta_R}}{\sqrt{2}}(R^{\pm})_{j7}+\frac{v_{\delta_R}}{\sqrt{2}}(R^{\pm})_{j8}\right],$$

$$a_{14,j}=\cos\theta_W\left[-\frac{\kappa_u}{2}(R^{\pm})_{j5}+\frac{\kappa_d}{2}(R^{\pm})_{j6}\right].$$

In addition, the gauge bosons interact with left and right scalar leptons and quarks: $(\tilde{\nu}_{Lk},\tilde{e}_{Lk})$ and $(\tilde{u}_{Lk},\tilde{d}_{Lk})$; $(\tilde{\nu}_{Rk},\tilde{e}_{Rk})$ and $(\tilde{u}_{Rk},\tilde{d}_{Rk})$. The Lagrangian density responsible for the interaction of $W_{L,R}$ bosons with scalar quarks and scalar leptons is

$$\mathcal{L}=\sum_m\left(i\frac{g_L}{2}\tilde{X}_{mn}\tilde{u}_{Lm}\overset{\leftrightarrow}{\partial^{\mu}}\tilde{d}_{Ln}W^{L+}_{\mu}+i\frac{g_L}{2}\tilde{Y}_{mn}\tilde{\nu}_{Lm}\overset{\leftrightarrow}{\partial^{\mu}}\tilde{e}_{Ln}W^{L-}_{\mu}\right)$$
$$+\text{H.c.}+\sum_m\left(i\frac{g_R}{2}\tilde{X}^*_{mn}\tilde{u}_{Rm}\overset{\leftrightarrow}{\partial^{\mu}}\tilde{d}_{Rn}W^{R+}_{\mu}\right.$$
$$\left.+i\frac{g_L}{2}\tilde{Y}^*_{mn}\tilde{\nu}_{Rm}\overset{\leftrightarrow}{\partial^{\mu}}\tilde{e}_{Rn}W^{R-}_{\mu}\right)+\text{H.c.} \tag{A17}$$

from which the couplings from Tables I–IV can be read off.

5. Interactions of vector bosons

The three-field interactions of vector bosons arise from the kinetic energy terms:

$$\mathcal{L}_{KE}=-\frac{1}{3}W^{La}_{\mu\nu}W^{L\mu\nu a}-\frac{1}{3}W^{Ra}_{\mu\nu}W^{R\mu\nu a}-\frac{1}{3}V_{\mu\nu}V^{\mu\nu} \tag{A18}$$

from which, after mixing, we can extract the relevant expressions for the three field interactions:

$$\mathcal{L}_{W_LW_LZ_L}=ig_L\cos\theta_W(W^{L+}_{\mu\nu}W^{L\mu-}Z^{L\nu}-W^{L-}_{\mu\nu}W^{L\mu+}Z^{L\nu}$$
$$+Z^L_{\mu\nu}W^{L\mu+}W^{L\nu-}),$$
$$\mathcal{L}_{W_LW_LA}=ig_L\sin\theta_W(W^{L+}_{\mu\nu}W^{L\mu-}A^{\nu}-W^{L-}_{\mu\nu}W^{L\mu+}A^{\nu}$$
$$+A_{\mu\nu}W^{L\mu+}W^{L\nu-}),$$
$$\mathcal{L}_{W_RW_RZ_L}=ig_R\sin\theta_W\sin\phi(W^{R+}_{\mu\nu}W^{R\mu-}Z^{L\nu}$$
$$-W^{R-}_{\mu\nu}W^{R\mu+}Z^{L\nu}+Z^L_{\mu\nu}W^{R\mu+}W^{R\nu-}), \tag{A19}$$
$$\mathcal{L}_{W_RW_RZ_R}=ig_R\cos\phi(W^{R+}_{\mu\nu}W^{R\mu-}Z^{R\nu}-W^{R-}_{\mu\nu}W^{R\mu+}Z^{R\nu}$$
$$+Z^R_{\mu\nu}W^{R\mu+}W^{R\nu-}),$$
$$\mathcal{L}_{W_RW_RA}=ig_R\cos\theta_W\sin\phi(W^{R+}_{\mu\nu}W^{R\mu-}A^{\nu}$$
$$-W^{R-}_{\mu\nu}W^{R\mu+}A^{\nu}+A_{\nu\mu}W^{R\mu+}W^{R\nu+}).$$

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